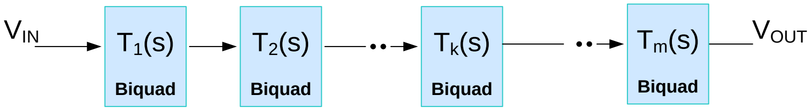
EE 508
Lecture 32

Leapfrog Networks

Review from last lecture

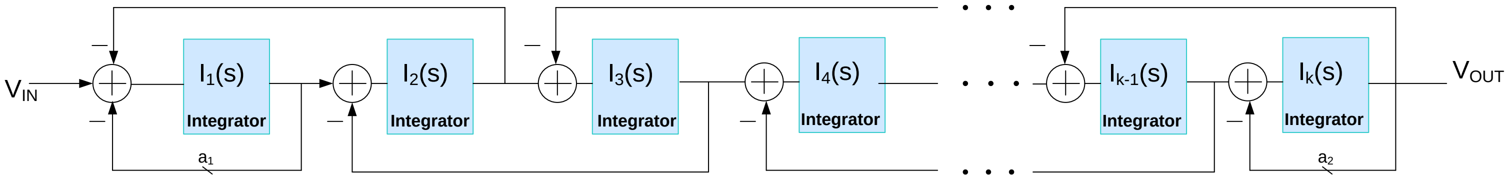
Filter Design/Synthesis Approaches

Cascaded Biquads

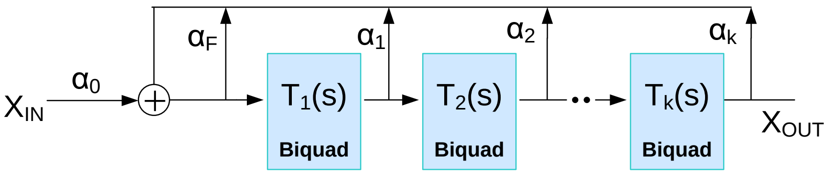


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog

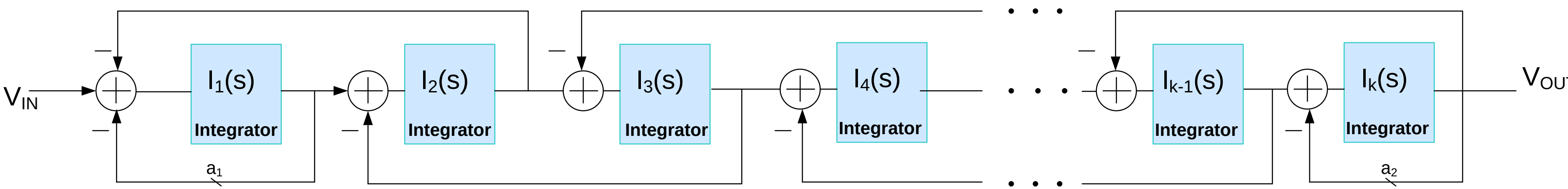


Multiple-loop Feedback – One type shown



Review from last lecture

Leapfrog Filters

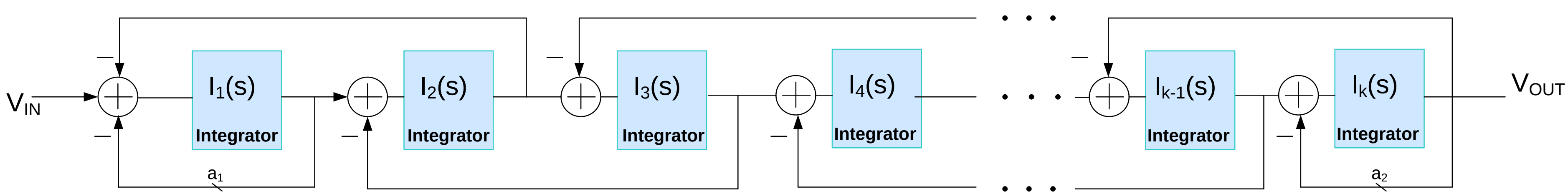


Introduced by Girling and Good, Wireless World, 1970

This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

Review from last lecture

Leapfrog Filters

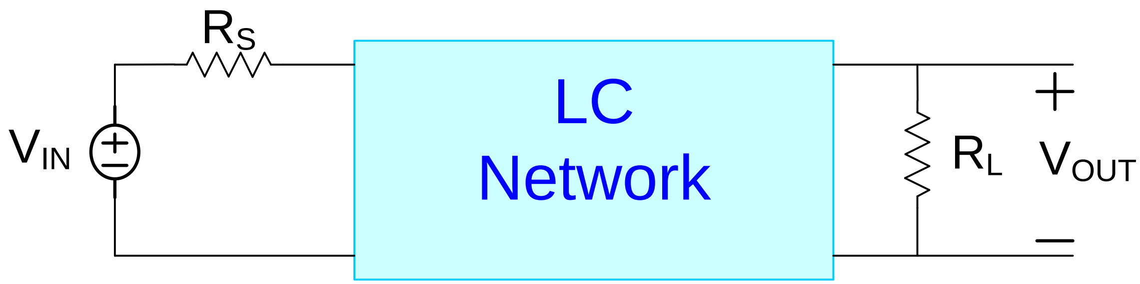


Observation: This structure appears to be dramatically different than anything else ever reported and it is not intuitive why this structure would serve as a filter, much less, have some unique and very attractive properties

To understand how the structure arose, why it has attractive properties, and to identify limitations, some mathematical background is necessary

Review from last lecture

Background Information for Leapfrog Filters



Assume the impedance R_S is fixed

Theorem 1: If the LC network delivers maximum power to the load at a frequency ω, then

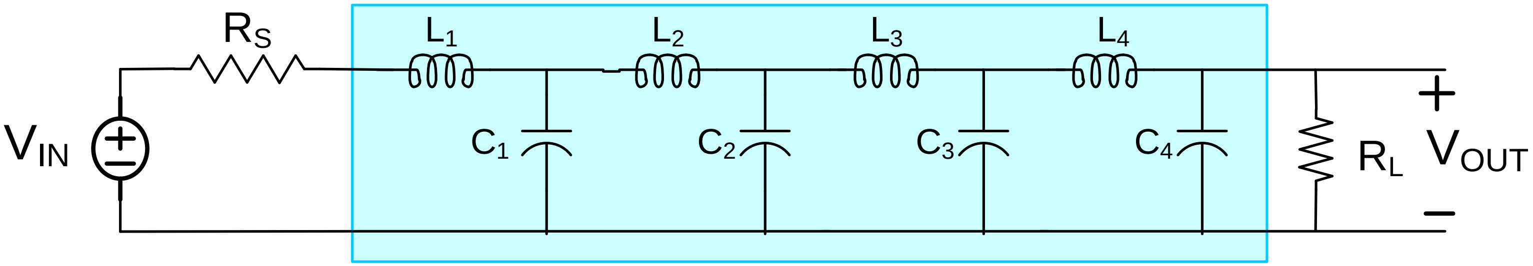
$$S_x^{|T(j\omega)|} = 0$$

for any circuit element in the system except for $x = R_L$

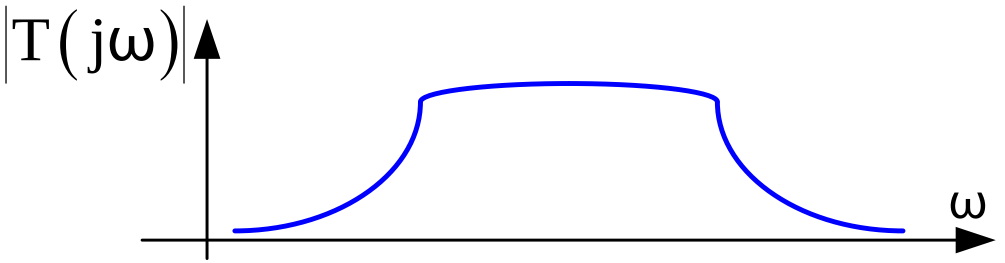
This theorem will be easy to prove after we prove the following theorem:

Implications of Theorem 1

Many passive LC filters such as that shown below exist that have near maximum power transfer in the passband



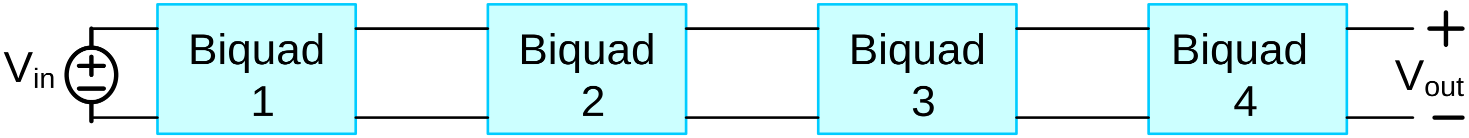
If a component in the LC network changes a little, there is little change in the passband gain characteristics (depicted as bandpass)



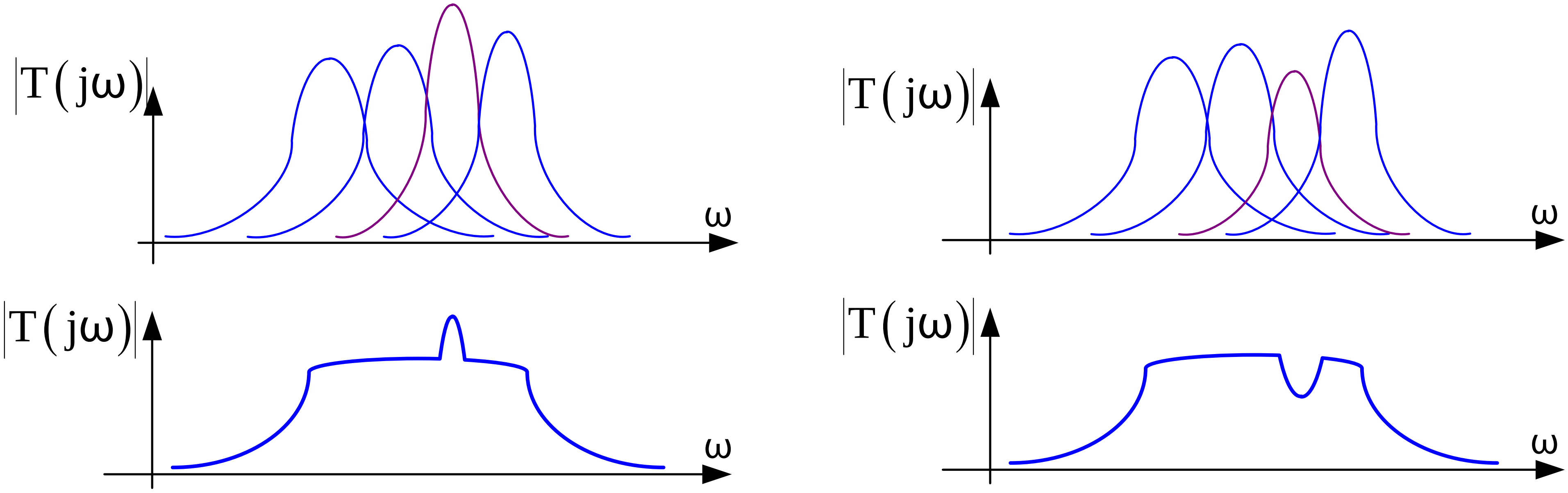
$$S_x^{|T(j\omega)|} \simeq 0 \qquad \text{in passband}$$

Review from last lecture

Implications of Theorem 1

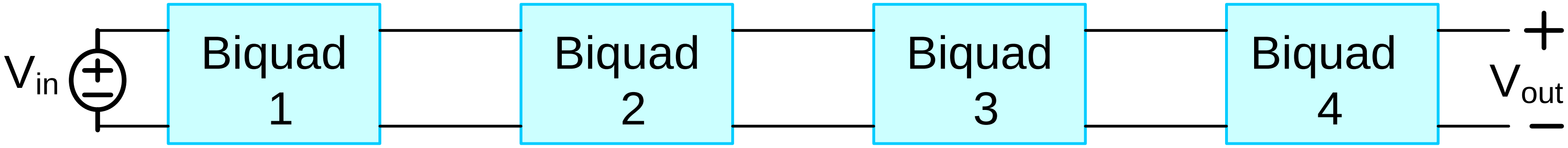


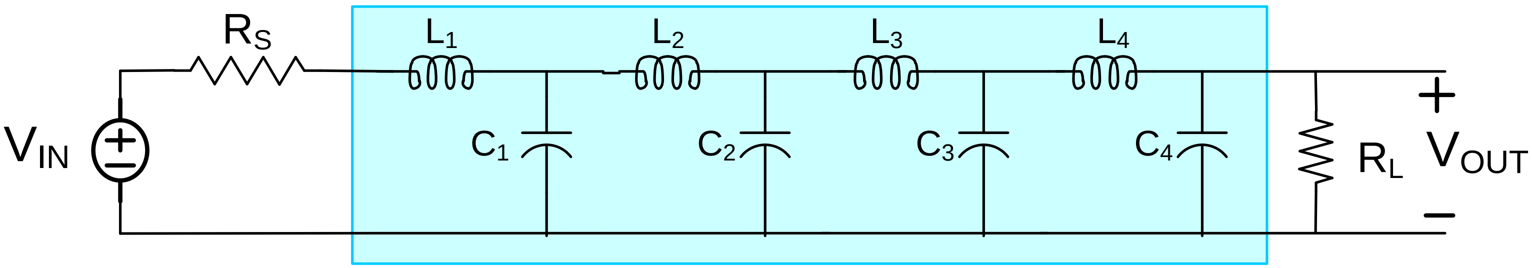
If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)



$$S_{x}^{|T(j\omega)|} \neq 0$$ in passband

Implications of Theorem 1

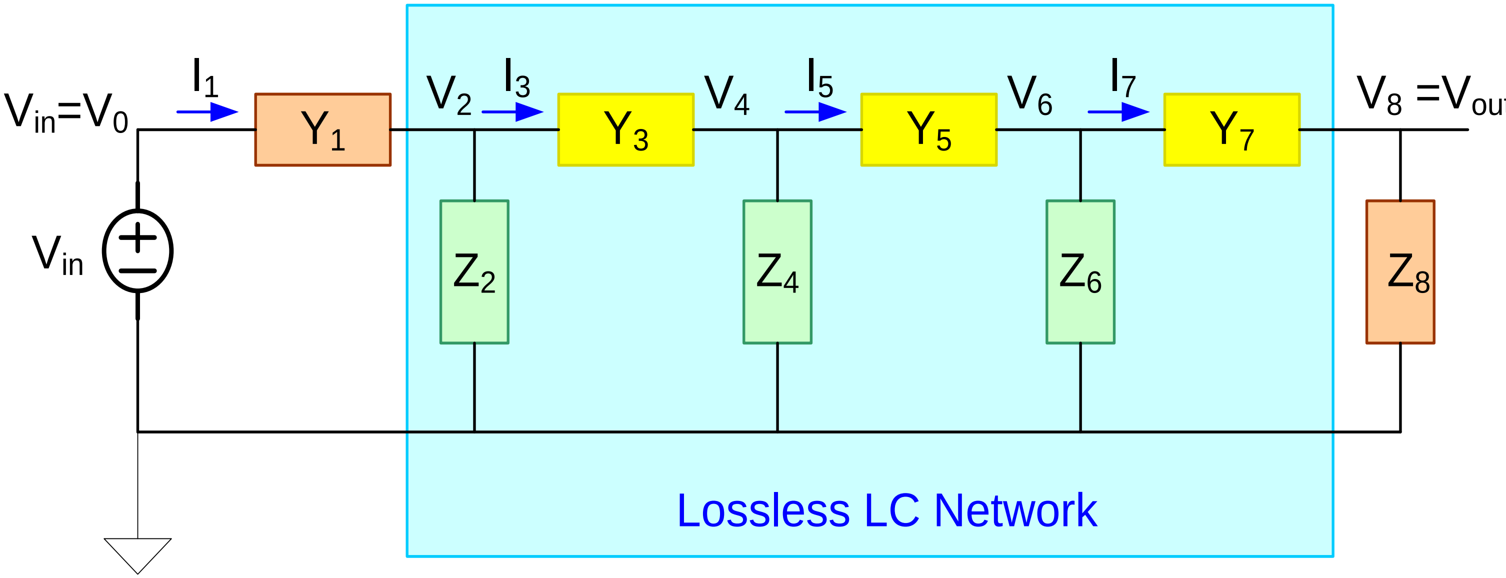




Good doubly-terminated LC networks often much less sensitive to most component values in the passband than are cascaded biquads !

This is a major advantage of the LC networks but can not be applied practically in most integrated applications or even in pc-board based designs

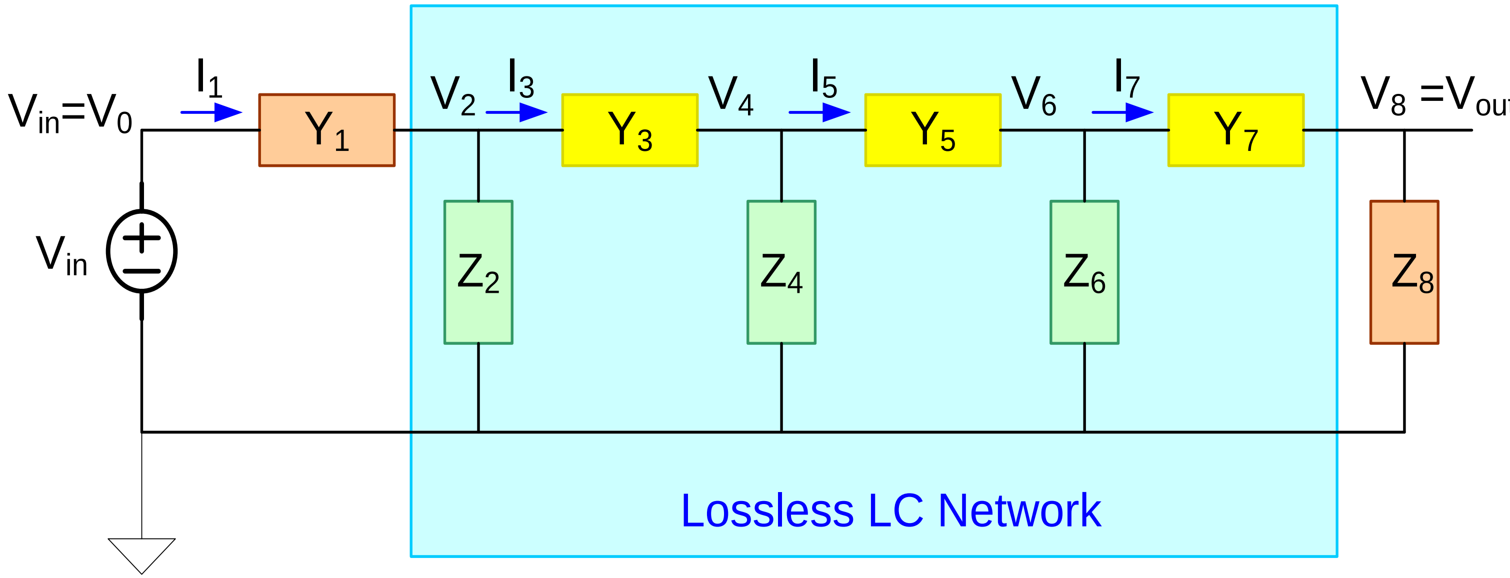
Doubly-terminated Ladder Network with Low Passband Sensitivities



For components in the LC Network observe

$$Y_k = \frac{1}{sL_k} \qquad\qquad Z_k = \frac{1}{sC_k}$$

Doubly-terminated Ladder Network with Low Passband Sensitivities



$$
\left.\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}\right\}
$$

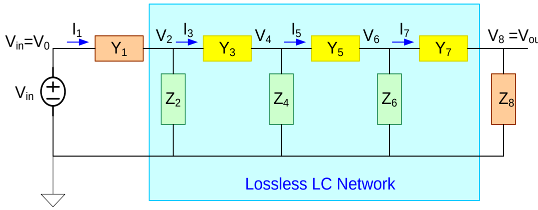
Complete set of independent equations that characterize this filter

Solution of this set of equations is tedious

All sensitivity properties of this circuit are inherently embedded in these equations!

Consider now only the set of equations and disassociate them from the circuit from where they came

$$
\left.
\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}
\right\}
$$



Make the associations

$$I_1 = V_1'$$

$$I_3 = V_3'$$

$$I_5 = V_5'$$

$$I_7 = V_7'$$

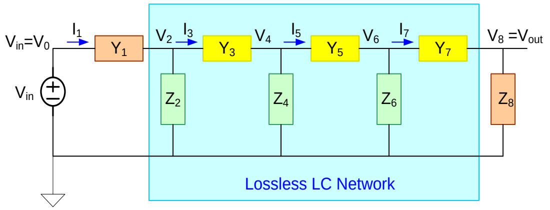
Rewrite the equations as

$$
\left.
\begin{aligned}
V_1' &= (V_0 - V_2)Y_1 \\
V_2 &= (V_1' - V_3')Z_2 \\
V_3' &= (V_2 - V_4)Y_3 \\
V_4 &= (V_3' - V_5')Z_4 \\
V_5' &= (V_4 - V_6)Y_5 \\
V_6 &= (V_5' - V_7')Z_6 \\
V_7' &= (V_6 - V_8)Y_7 \\
V_8 &= V_7' Z_8
\end{aligned}
\right\}
$$

This association is nothing more than a renaming of variables so all sensitivities WRT Y's and Z's will remain unchanged!

Consider now only the set of equations and disassociate them from the circuit from where they came

$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)Y_1 \\
V_2 &= (V_1' - V_3')Z_2 \\
V_3' &= (V_2 - V_4)Y_3 \\
V_4 &= (V_3' - V_5')Z_4 \\
V_5' &= (V_4 - V_6)Y_5 \\
V_6 &= (V_5' - V_7')Z_6 \\
V_7' &= (V_6 - V_8)Y_7 \\
V_8 &= V_7' Z_8
\end{aligned}\right\}$$



For the LC filter, recall

$$Y_k = \frac{1}{sL_k} \qquad Z_k = \frac{1}{sC_k}$$

And the source and load termination relationships were

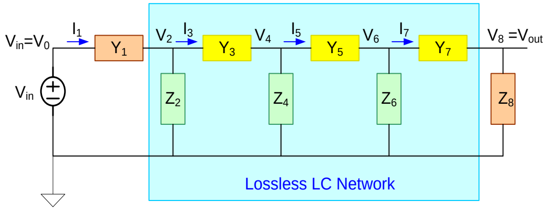
$$Y_1 = \frac{1}{R_1} \qquad Z_8 = R_8$$

These can be written as

$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5} \\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6} \\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7} \\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$

Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

Consider now only the set of equations and disassociate them from the circuit from where they came



$$V_1' = (V_0 - V_2)\frac{1}{R_1}$$

$$V_2 = (V_1' - V_3')\frac{1}{sC_2}$$

$$V_3' = (V_2 - V_4)\frac{1}{sL_3}$$

$$V_4 = (V_3' - V_5')\frac{1}{sC_4}$$

$$V_5' = (V_4 - V_6)\frac{1}{sL_5}$$

$$V_6 = (V_5' - V_7')\frac{1}{sC_6}$$

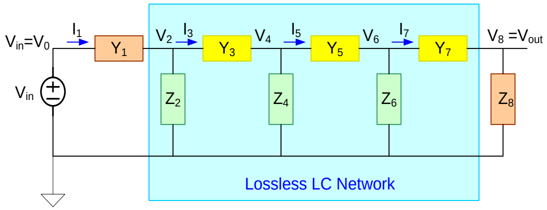
$$V_7' = (V_6 - V_8)\frac{1}{sL_7}$$

$$V_8 = V_7' R_8$$

Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

If any circuit is characterized by these equations, the sensitivities to the integrator gains will be identical to the sensitiviies of the original circuit to the Ls and Cs !

Consider now only the set of equations and disassociate them from the circuit from where they came



$$V_1' = (V_0 - V_2)\frac{1}{R_1}$$

$$V_2 = (V_1' - V_3')\frac{1}{sC_2}$$

$$V_3' = (V_2 - V_4)\frac{1}{sL_3}$$

$$V_4 = (V_3' - V_5')\frac{1}{sC_4}$$

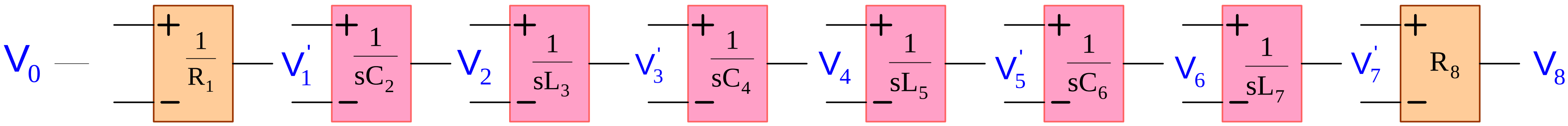
$$V_5' = (V_4 - V_6)\frac{1}{sL_5}$$

$$V_6 = (V_5' - V_7')\frac{1}{sC_6}$$

$$V_7' = (V_6 - V_8)\frac{1}{sL_7}$$

$$V_8 = V_7' R_8$$

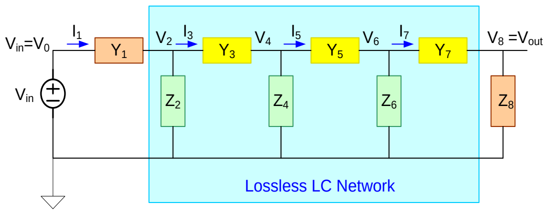
Each equation corresponds to either an integrator or summer with the output voltage output variables and the gain indicated (don’t worry about the units)



$$V_0 = V_{in}$$

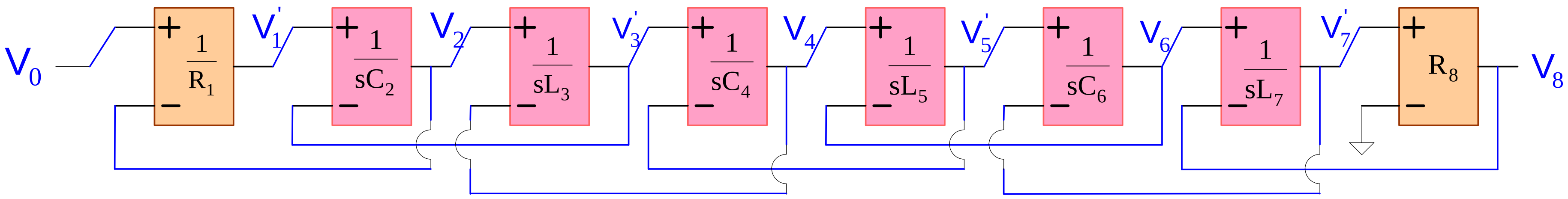
$$V_8 = V_{out}$$

Consider now only the set of equations and disassociate them from the circuit from where they came

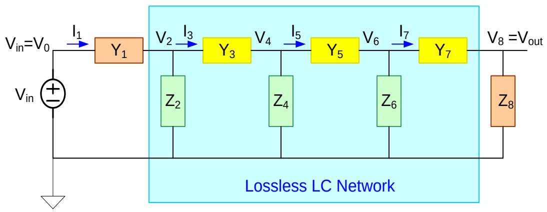


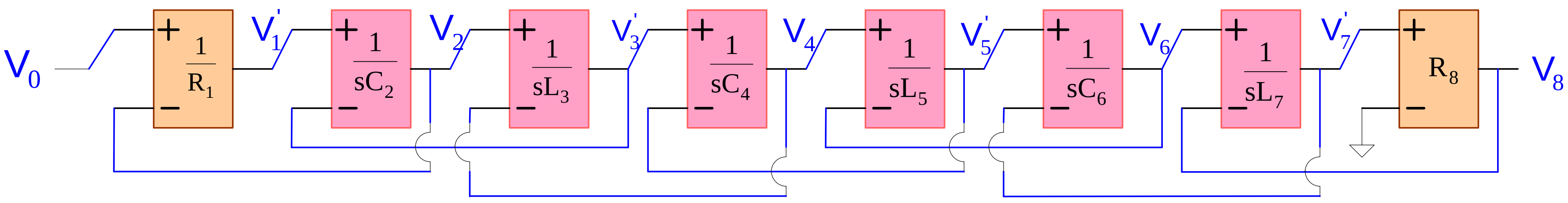
$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & \qquad V_5' &= (V_4 - V_6)\frac{1}{sL_5}\\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6}\\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7}\\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$

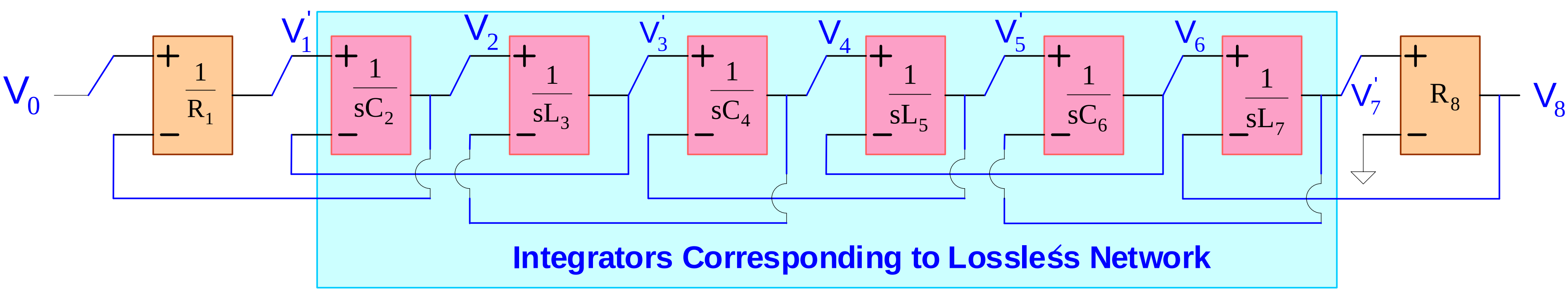
The interconnections that complete each equation can now be added



Consider now only the set of equations and disassociate them from the circuit from where they came



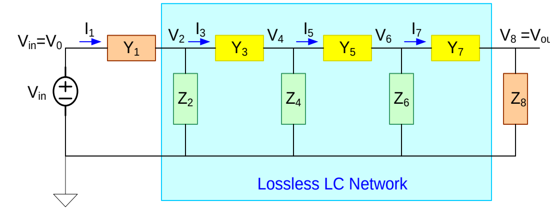


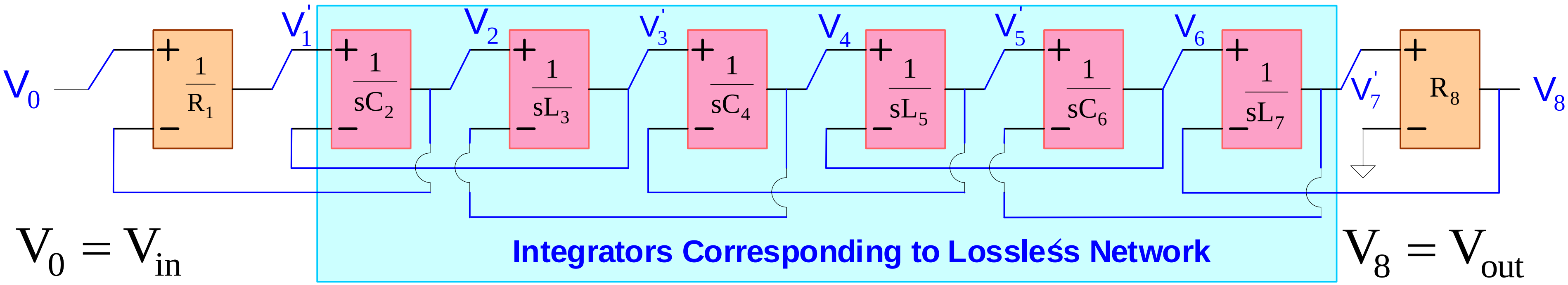


$$V_0 = V_{in}$$

$$V_8 = V_{out}$$

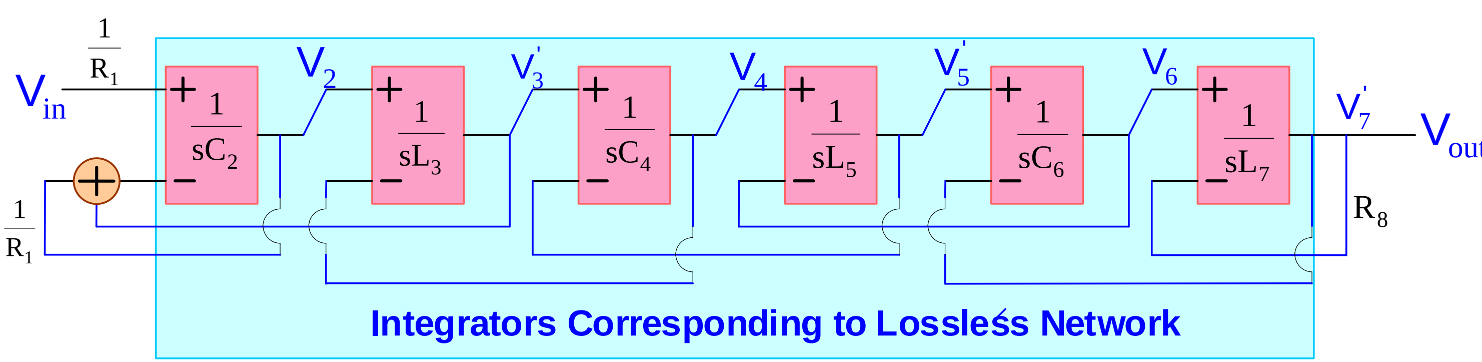
The Leapfrog Configuration



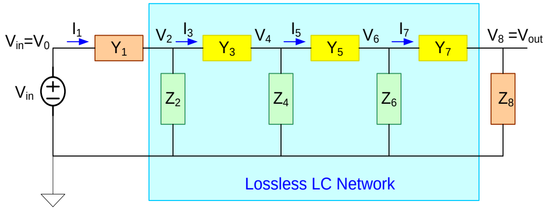


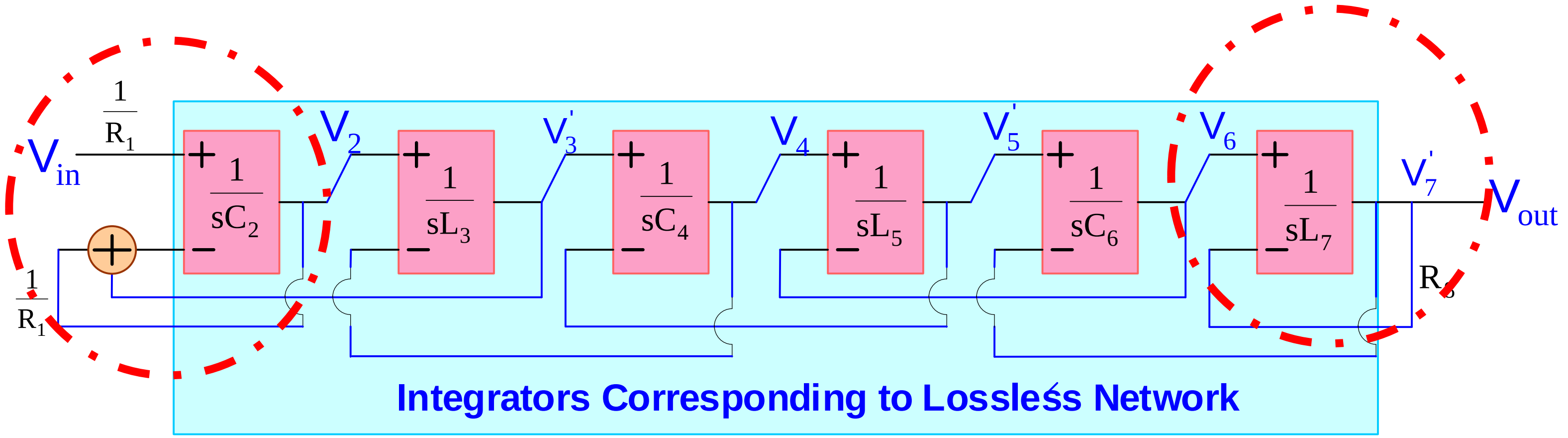
Input summing and weighting can occur at input to the first integrator

The difference between V_8 and V'_7 is only a scale factor that does not affect shape, and the weighting on the Vin input also does not affect shape, thus



The Leapfrog Configuration

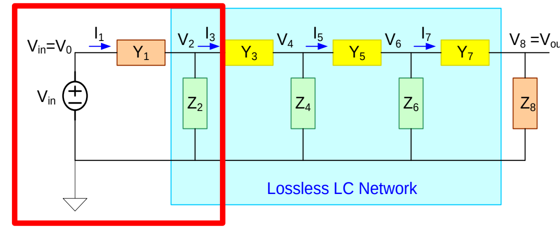


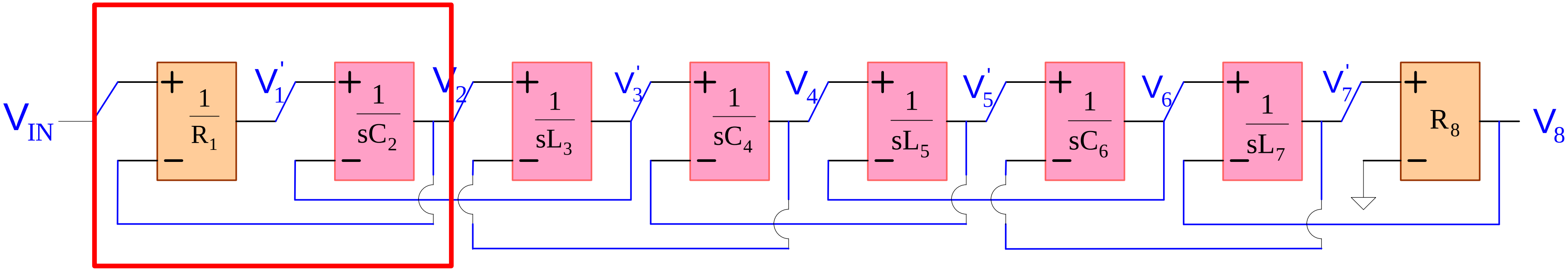


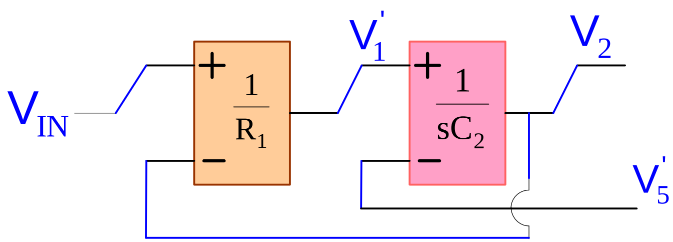
The terminations on both sides have local feedback around an integrator which can be alternately viewed as a lossy integrator

Could redraw the structure as a cascade of internal lossless integrators with terminations that are lossy integrators but since there are so many different ways to implement the integrators and summers, we will not attempt to make that association in the block diagram form but in most practical applications a lossy integrator is often used on the input or the output or both

Consider the first two stages:







$$V_1' = (V_0 - V_2)\frac{1}{R_1}$$

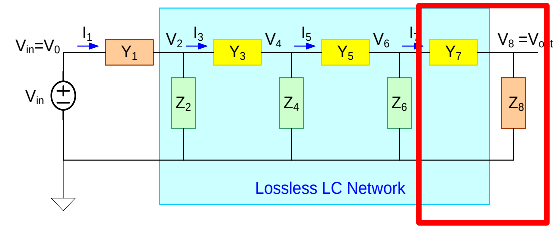
$$V_2 = (V_1' - V_3')\frac{1}{sC_2}$$

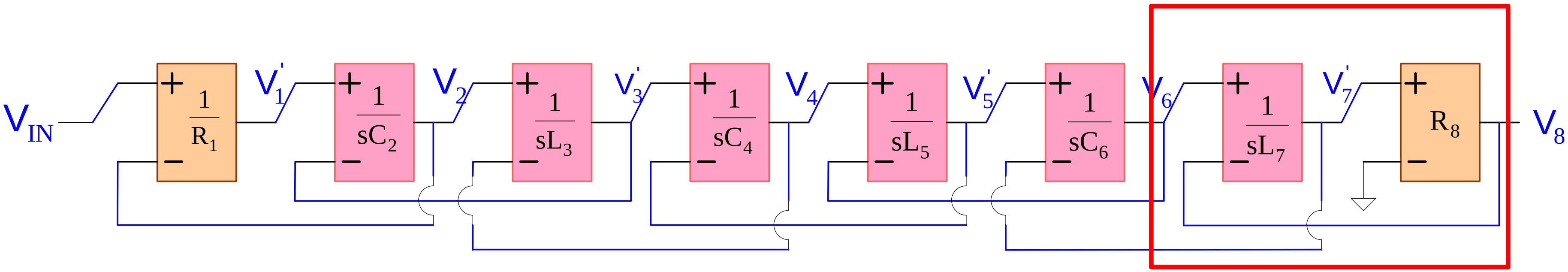
$$V_2 = \left((V_0 - V_2)\frac{1}{R_1} - V_3'\right)\frac{1}{sC_2}$$

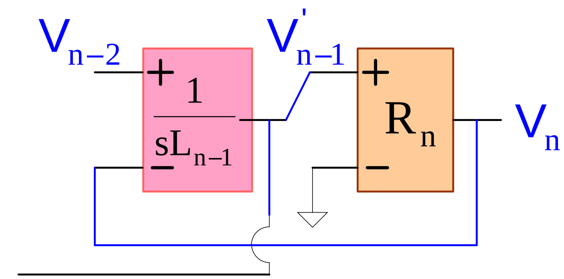
$$V_2 = V_{IN}\left(\frac{1}{1+R_1C_2s}\right) - V_3'\left(\frac{R_1}{1+R_1C_2s}\right)$$

These two blocks act as a single summing lossy integrator block with loss factor R_1

Consider the last two stages:







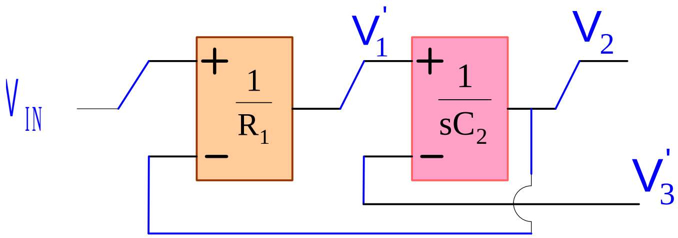
$$\left.\begin{aligned} V'_{n-1} &= (V_{n-2} - V_n)\frac{1}{sL_{n-1}} \\ V_n &= V'_{n-1} R_n \end{aligned}\right\}$$

$$V_n = (V_{n-2} - V_n)\frac{1}{sL_{n-1}} R_n$$

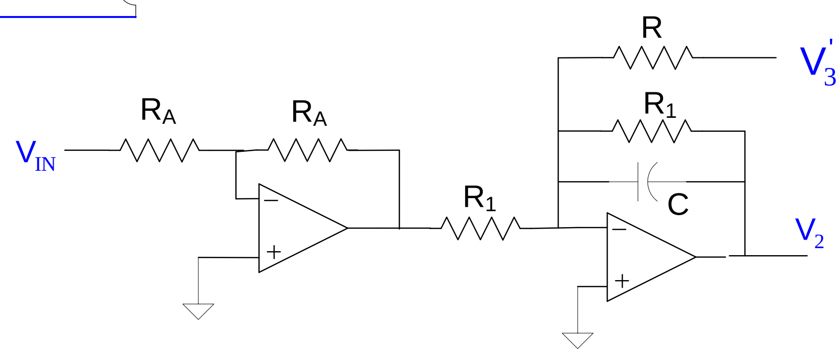
$$V_n = V_{n-2}\left(\frac{R_n}{sL_{n-1} + R_n}\right)$$

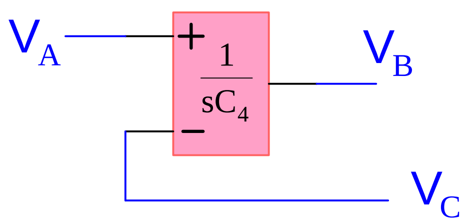
These two blocks act as a lossy integrator block with loss factor R_n

Implementation with Miller Integrators:

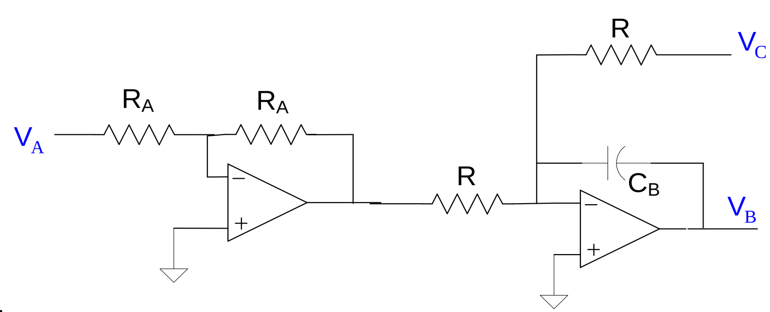


$$V_2 = V_{IN}\left(\frac{1}{1+R_1C_2s}\right) - V_3'\left(\frac{R_1}{1+R_1C_2s}\right)$$



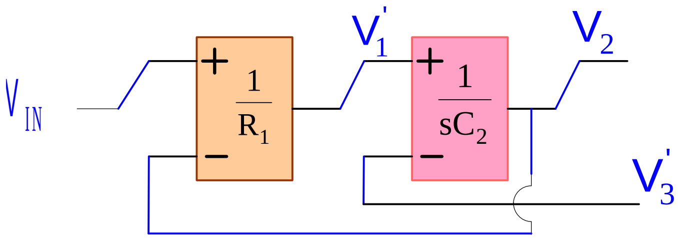


$$V_B = V_A\left(\frac{1}{RC_Bs}\right) - V_C\left(\frac{1}{1+RC_Bs}\right)$$

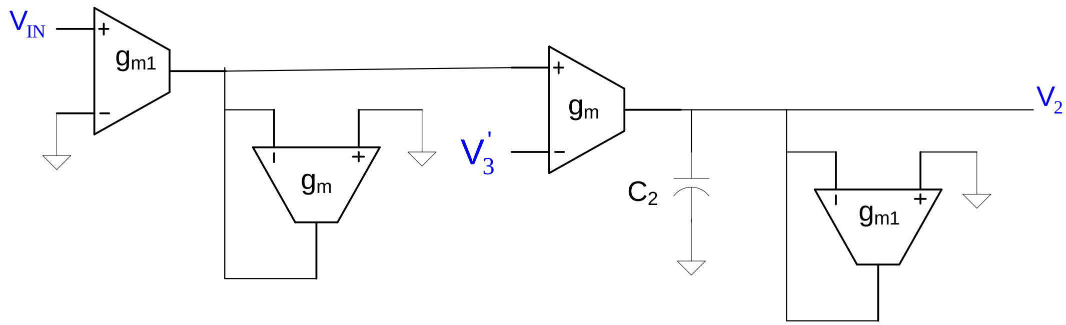


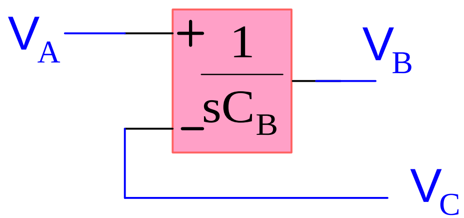
Can fix either R or C on each stage

Implementation with OTA-C Integrators:

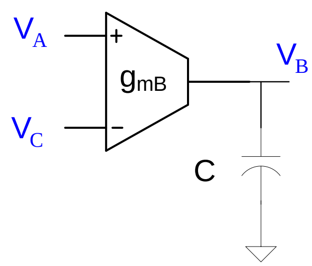


$$V_2 = V_{IN}\left(\frac{1}{1+R_1C_2s}\right) - V_3'\left(\frac{R_1}{1+R_1C_2s}\right)$$



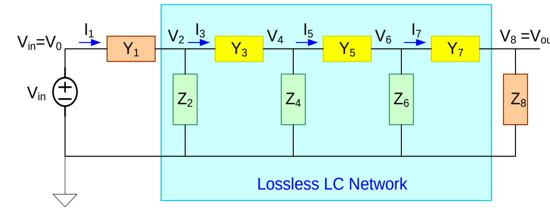


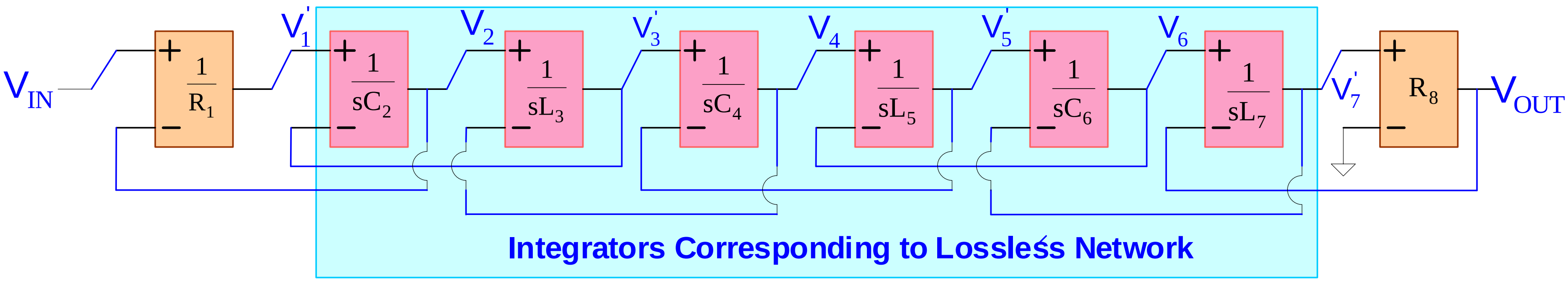
$$V_B = V_A\left(\frac{1}{RC_Bs}\right) - V_C\left(\frac{1}{1+RC_Bs}\right)$$



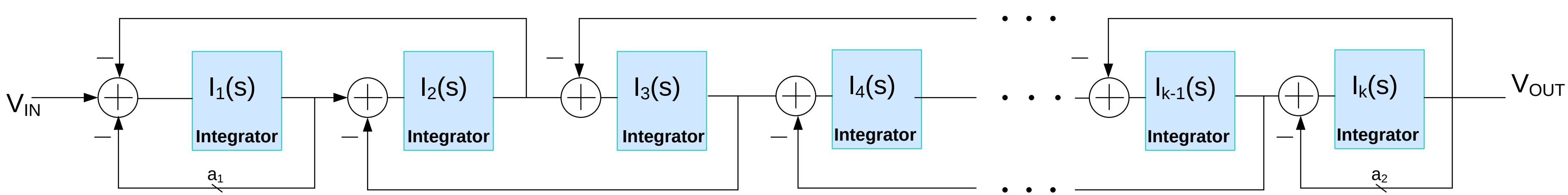
Can fix either g_m or C on each stage

The Leapfrog Configuration



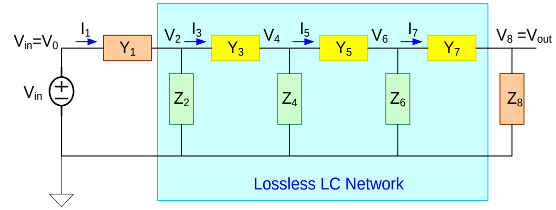


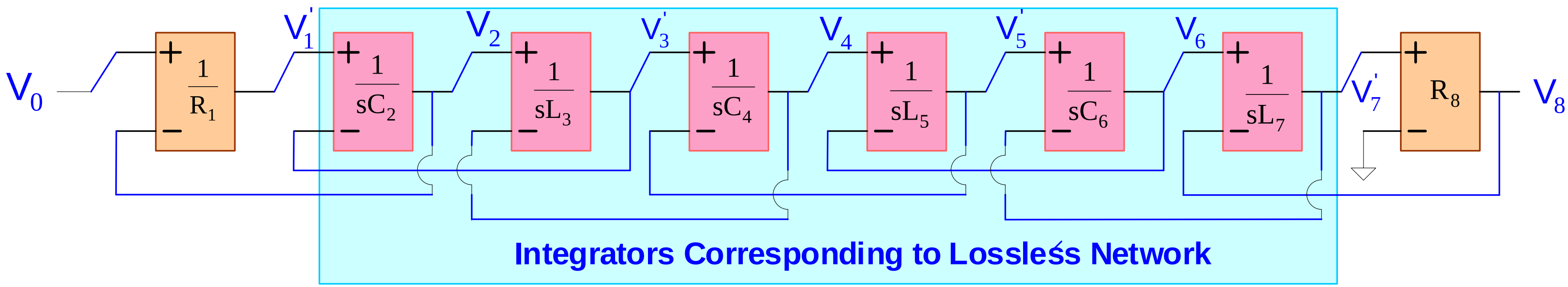
In the general case, this can be redrawn as shown below



Note the first and last integrators become lossy because of the local feedback

The Leapfrog Configuration





The passive prototype filter from which the leapfrog was designed has all shunt capacitors and all series inductors and is thus lowpass.

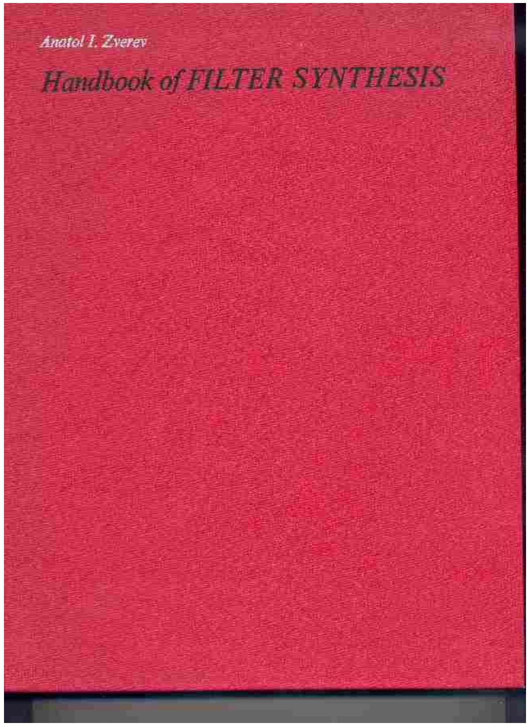
The resultant leapfrog filter has the same transfer function and is thus lowpass

The Passive Prototypes with Maximum Power Transfer in Passband

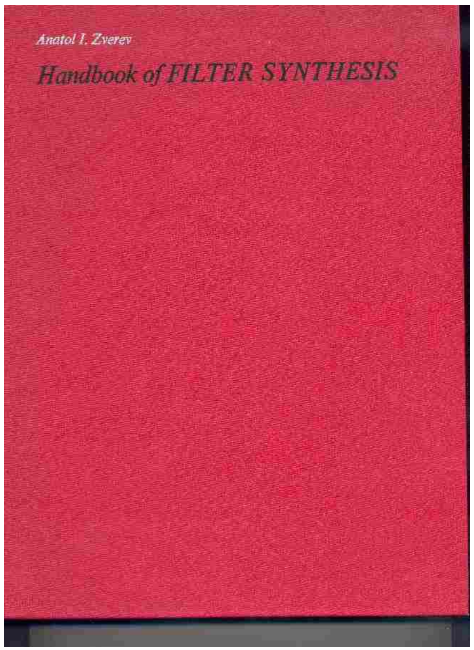
Doubly-terminated LC filters with near maximum power transfer in the passband were developed from the 30's to the 60's

Seldom discussed in current texts but older texts and occasionally software tools provide the passive structures needed to synthesize leapfrog networks

One good book is that by Zverev

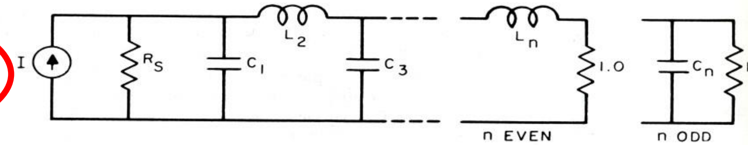


The Passive Prototypes with Maximum Power Transfer in Passband

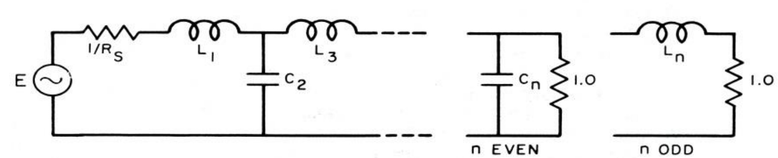


LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE



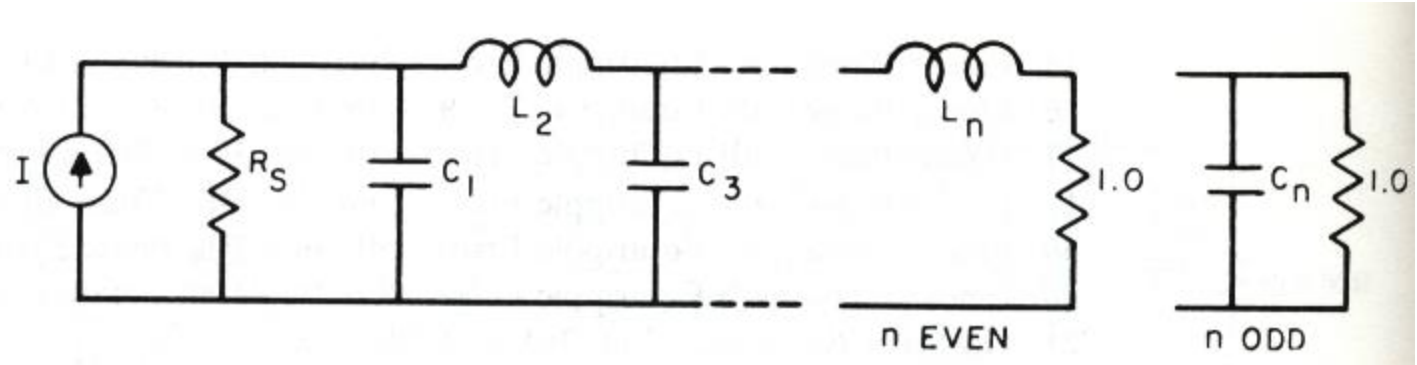
n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.8352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3882	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2690	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4



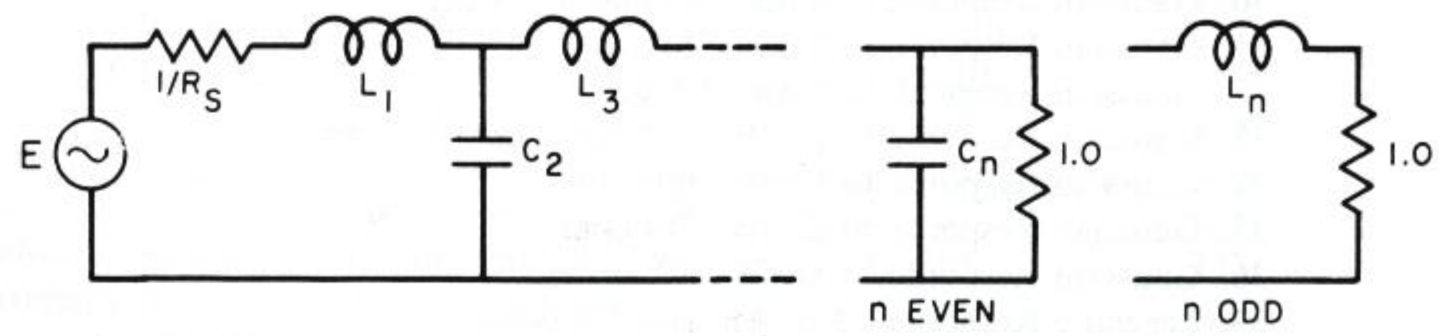
Must start with correct filter type:
(e.g. BW, CC, Cauer)

The Passive Prototypes with Maximum Power Transfer in Passband

The Butterworth Low-Pass Filters



Loading element is a shunt capacitor (appear from top to bottom in table)



Loading element is a series inductor (appear from bottom to top in table)

Can do Thevenin-Norton Transformations

The Passive Prototypes with Maximum Power Transfer in Passband

Normalized so R_L=1

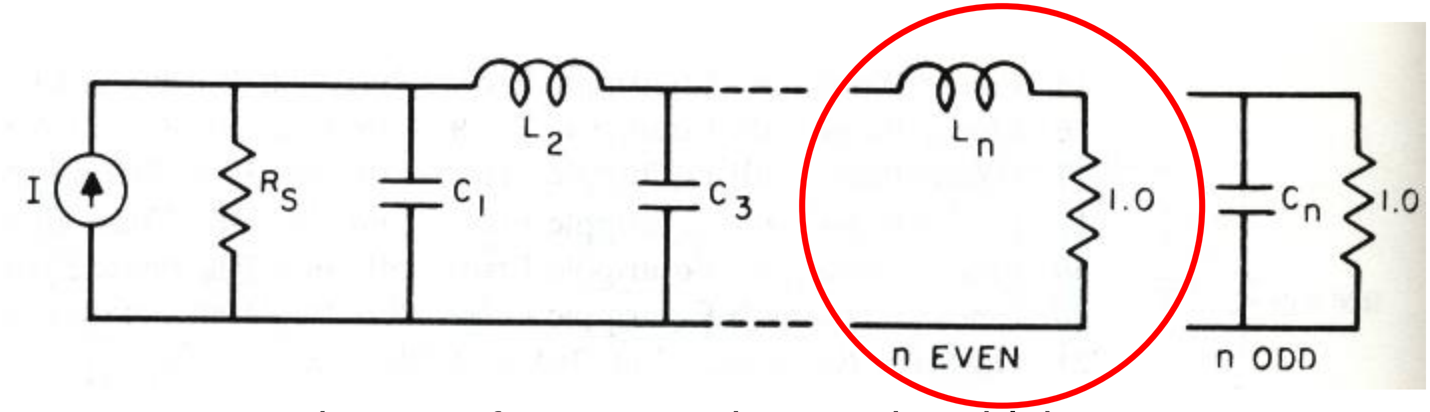
n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.8352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3882	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2690	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	INF.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	INF.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	INF.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

Example:

Design a 6th-order BW lowpass Leapfrog filter with equal source and load terminations, and with a 3dB band edge of 4KHz.

Start with the normalized BW lowpass filter



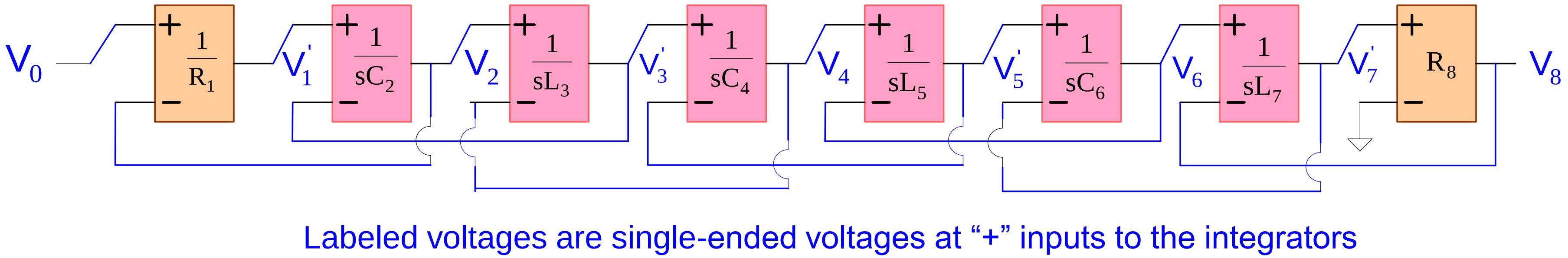
(appear from top to bottom in table)

Do Norton to Thevenin transformation at input

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	INF.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	INF.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	INF.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

R_s=1, C_1=.5176, L_2=1.414, C_3=1.939, L_4=1.9319, C_5=1.4142, L_6=0.5176

Note index differs by 1 from that used for Leapfrog formulation



Changing the index notation:

$R_1=1$, $C_2=.5176$, $L_3=1.414$, $C_4=1.939$, $L_5=1.9319$, $C_6=1.4142$, $L_7=0.5176$

Implement in the technology of choice

Combine loss on input and output integrators to eliminate two stages

Do frequency denormalization to obtain band-edge at 4KHz

Do impedance scaling to obtain acceptable component values

Bandpass Leapfrog Structures

Consider lowpass to bandpass transformations

Un-normalized

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

Normalized

$$s_n \rightarrow \frac{s^2 + 1}{sBW_n}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW_n}{s^2 + 1}$$

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW_n}{s^2 + s\alpha BW_n + 1}$$

Bandpass Leapfrog Structures

Bandpass Leapfrog Structure obtained by replacing integrators by the corresponding transformed block

Zero sensitivity to parameters in the transformed blocks will be retained at the image frequencies of the bandpass filter

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$

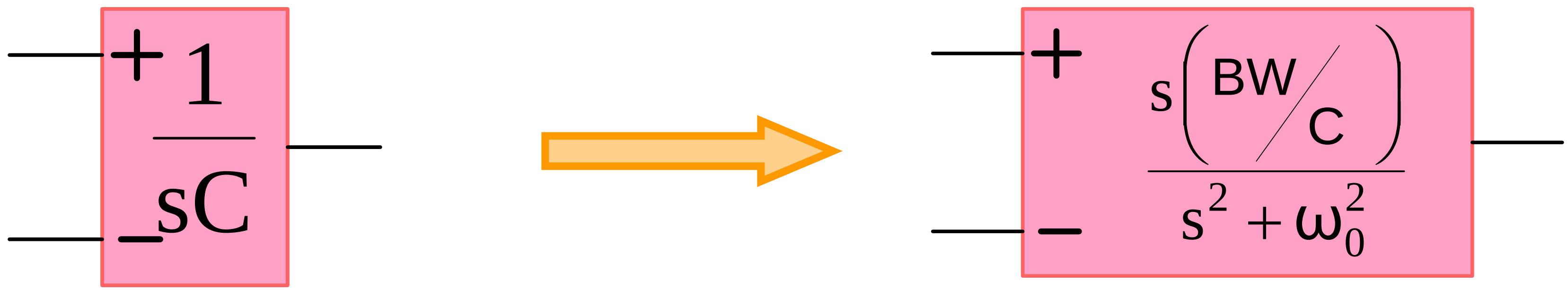
Integrators map to bandpass biquads with infinite Q

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

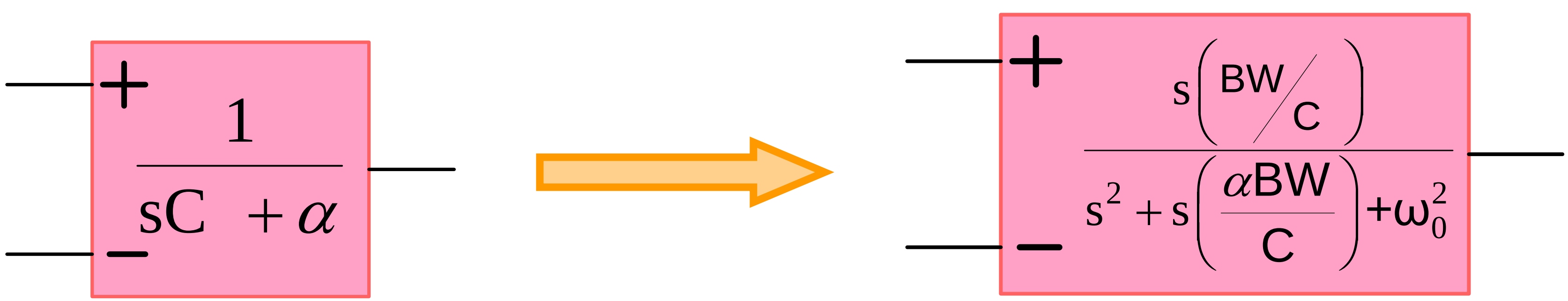
Lossy integrators map to bandpass biquads with finite Q

Bandpass Leapfrog Structures

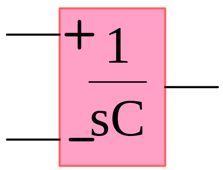
$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$



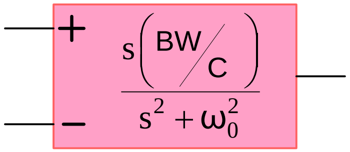
$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

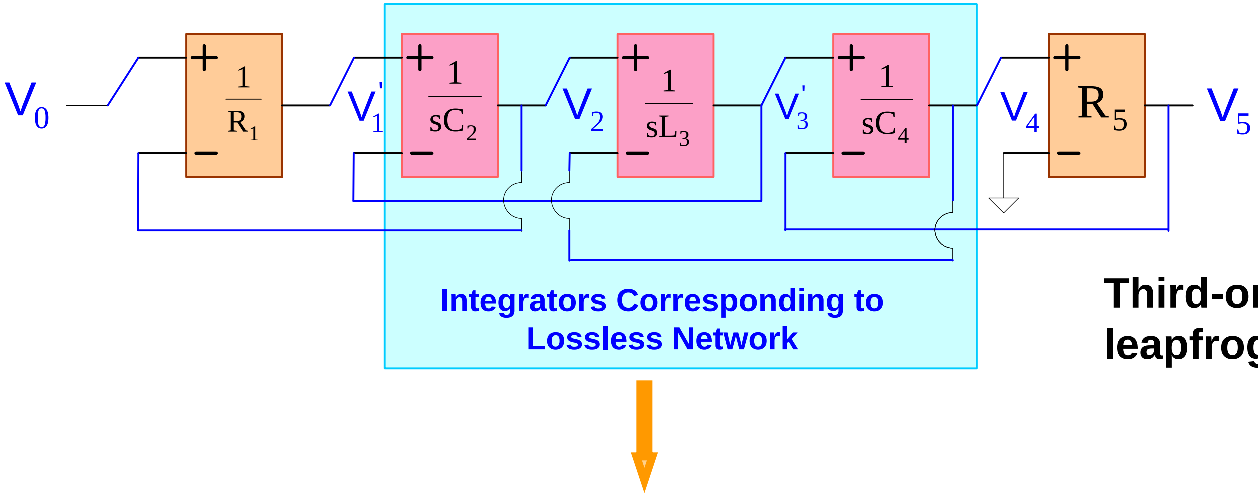


Bandpass Leapfrog Structures

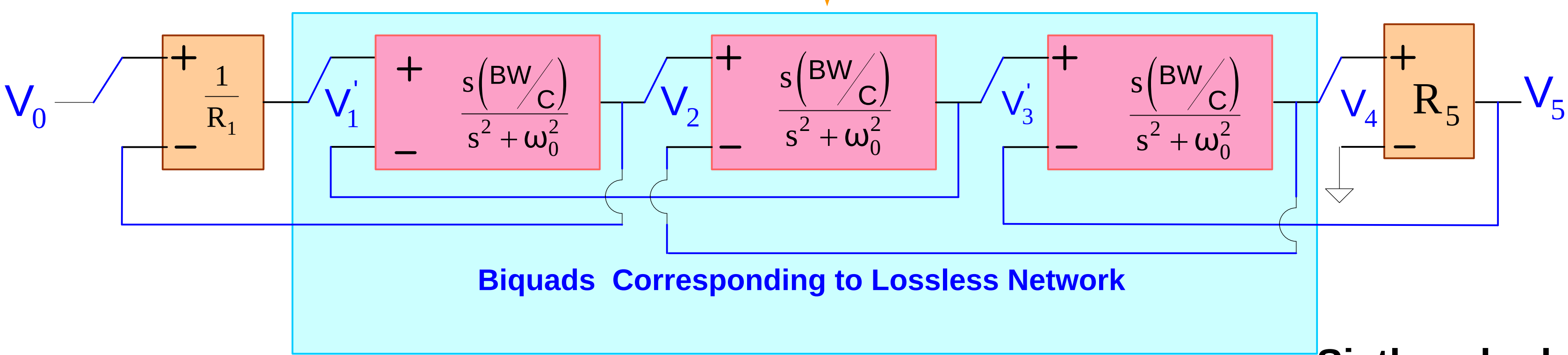


$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$





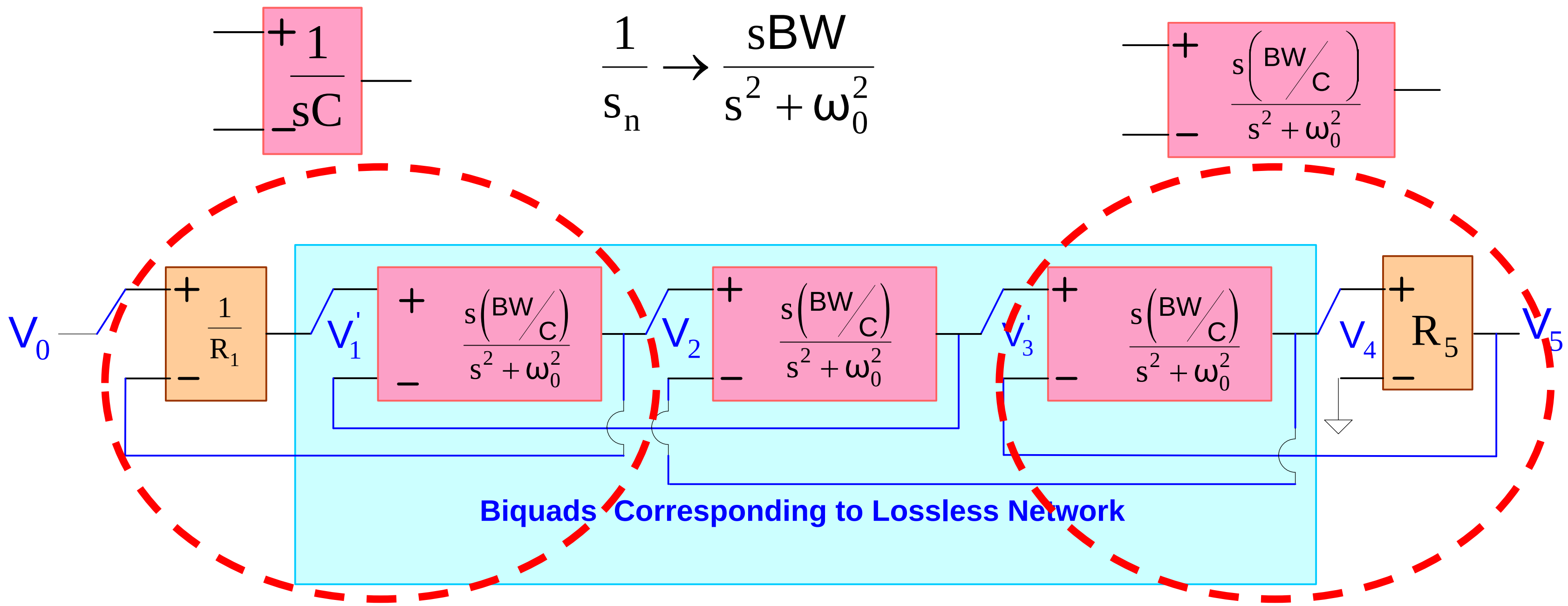
Third-order lowpass leapfrog filter



Sixth-order bandpass leapfrog filter

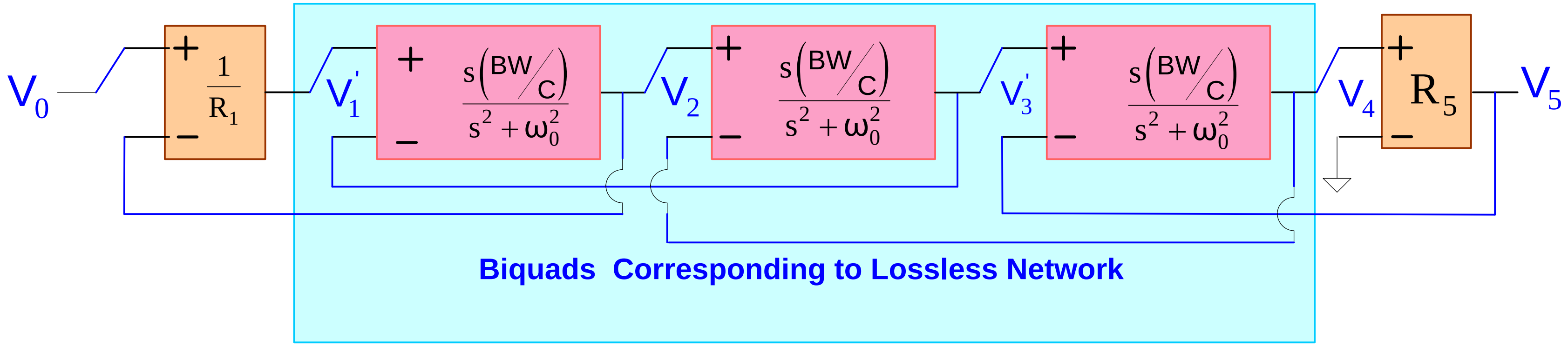
Bandpass Leapfrog Structures

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$



"Loss" at input and/or output can usually be incorporated into finite-Q terminating biquads instead of requiring additional voltage amplifiers

Bandpass Leapfrog Structures



- The bandpass biquads can be implemented with various architectures and the architecture does not ideally affect the passband sensitivity of the filter

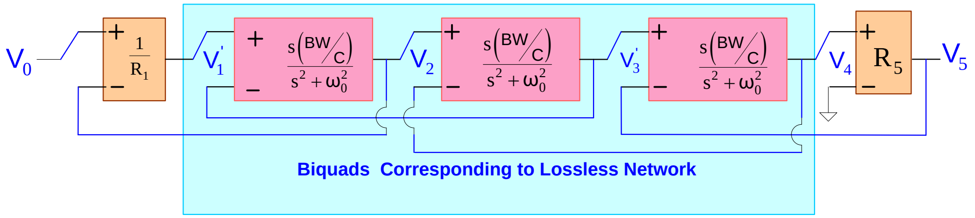
- Integrator-based biquads are often used in integrated applications

- Note the lossless biquads are infinite Q structures !

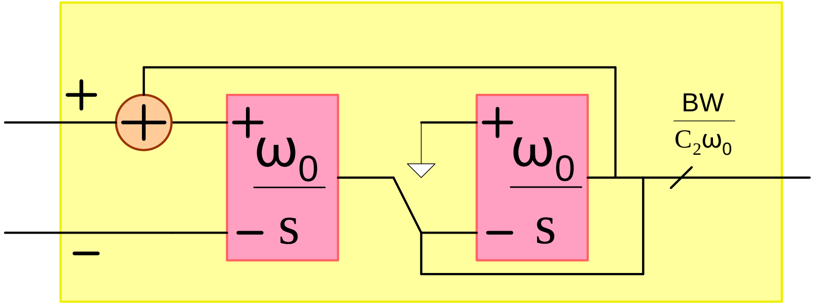
Is it easy or practical to implement infinite Q biquads?

Are there stability concerns about the infinite Q biquads?

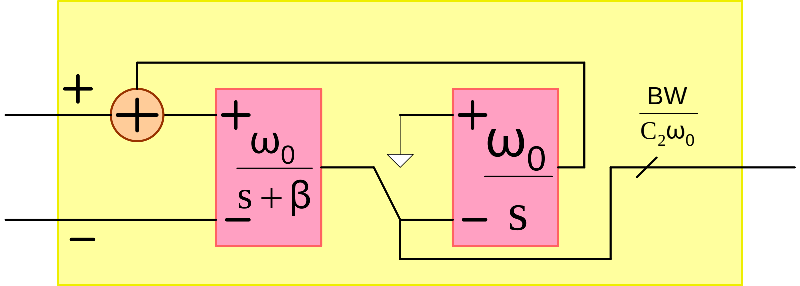
Bandpass Leapfrog Structures



Integrator-based biquads



$$T(s) = \frac{s(BW/C)}{s^2 + \omega_0^2}$$

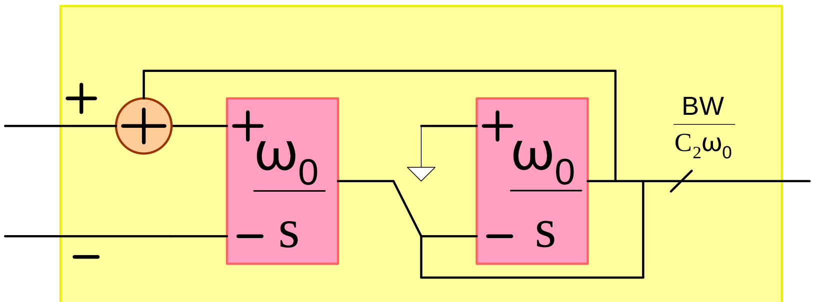


$$T(s) = \frac{s(BW/C)}{s^2 + s\alpha BW + \omega_0^2}$$

Bandpass Leapfrog Structures

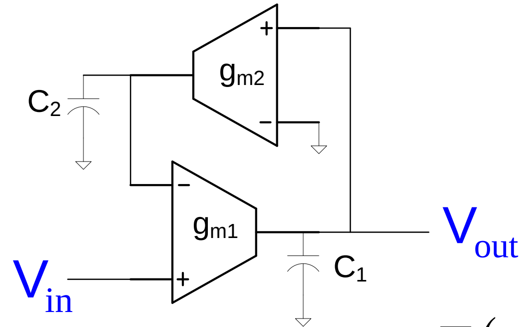
Integrator-based biquads

Infinite Q bandpass biquad



$$T(s) = \frac{s\left(BW/C\right)}{s^2 + \omega_0^2}$$

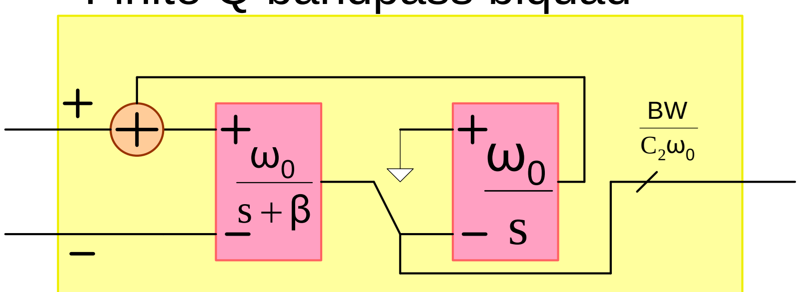
OTA-C Implementations (Concept)



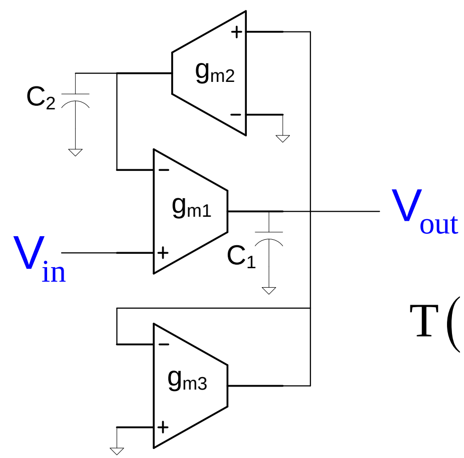
(Not Differential)

$$T(s) = \frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

Finite Q bandpass biquad



$$T(s) = \frac{s\left(BW/C\right)}{s^2 + s\alpha BW + \omega_0^2}$$



(Not Differential)

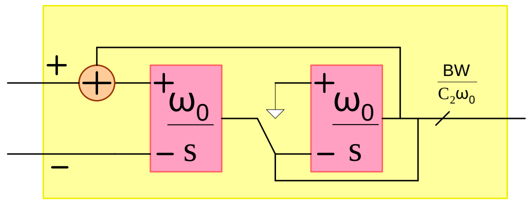
$$T(s) = \frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2 + s\frac{g_{m3}}{C_1} + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

Bandpass Leapfrog Structures

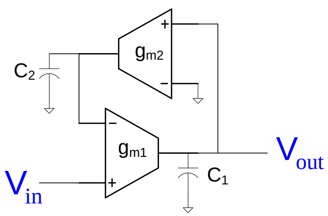
Integrator-based biquads

OTA-C Implementations

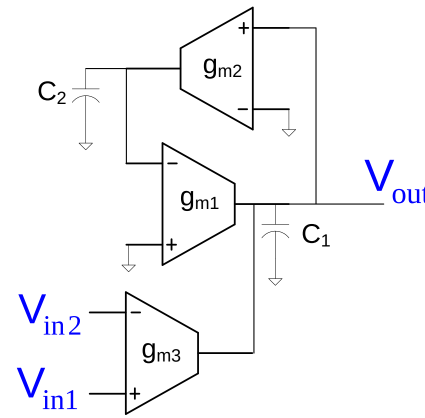
Infinite Q bandpass biquad



$$T(s) = \frac{s(BW/C)}{s^2 + \omega_0^2}$$



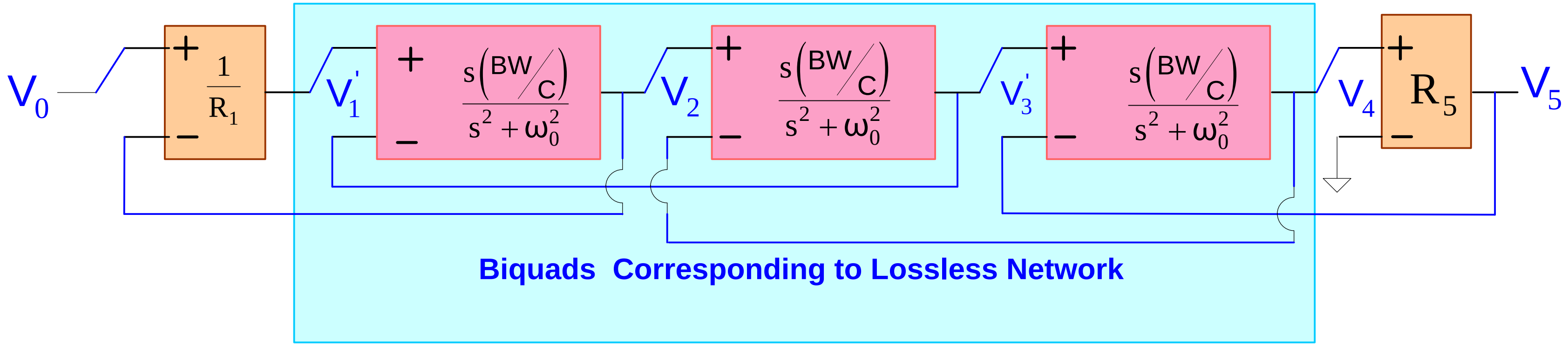
$$T(s) = \frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}}$$



$$V_{OUT}(s) = \frac{s\left(\frac{g_{m3}}{C_1}\right)[V_{in1} - V_{in2}]}{s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

Multiple inputs can be added to lossy integrator too!

Bandpass Leapfrog Structures



Note the lossless biquads are infinite Q structures !

Is it easy or practical to implement infinite Q biquads?

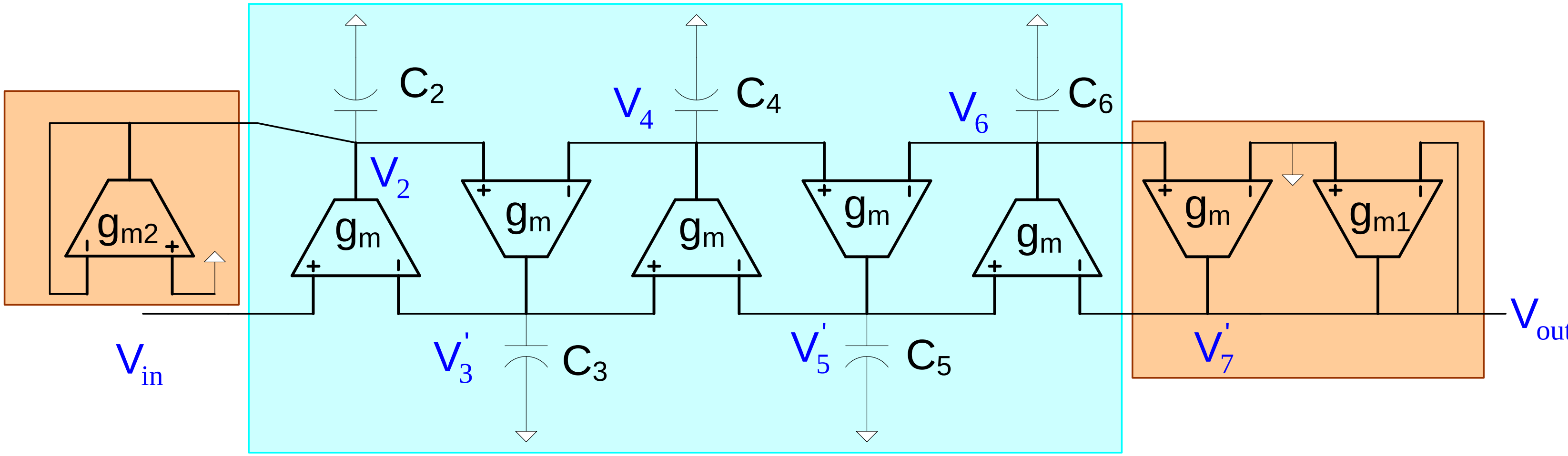
Yes – have shown by example in g_m-C family and also easy in other families

Are there stability concerns about the infinite Q biquads?

Stability of overall leapfrog structure of concern, not stability of individual biquads
Overall leapfrog structure is robust with low passband sensitivities !

Leapfrog Implementations

Fifth-order Lowpass Leapfrog with OTAs



$$V_1' = \frac{1}{R_1}\left(V_{in} - V_2\right)$$

$$V_2 = \frac{g_m / C_2}{s}\left(V_1' - V_3'\right)$$

$$V_3' = \frac{g_m / C_3}{s}\left(V_2 - V_4\right)$$

$$V_4 = \frac{g_m / C_4}{s}\left(V_3' - V_5'\right)$$

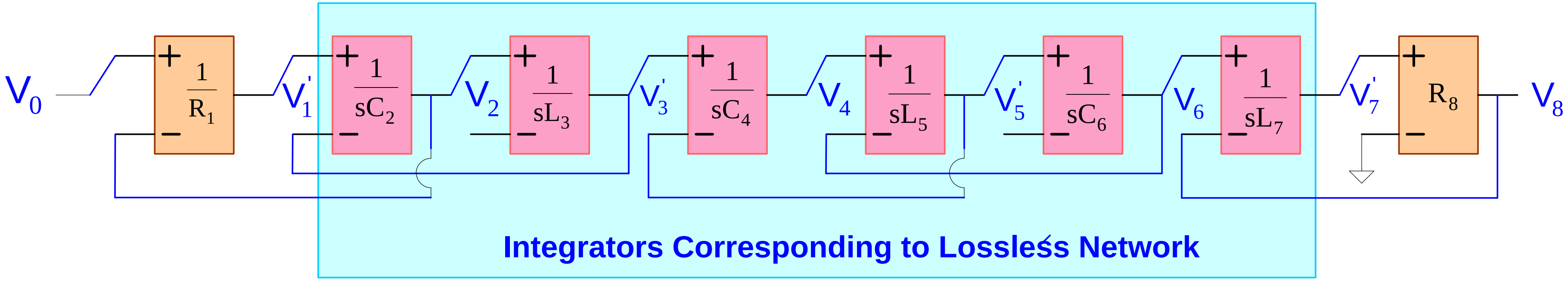
$$V_5' = \frac{g_m / C_5}{s}\left(V_4 - V_6\right)$$

$$V_6 = \frac{g_m / C_6}{s}\left(V_5 - V_7'\right)$$

$$V_7' = \left(g_m / g_{m1}\right)V_6$$

Practically can either fix g_ms and vary capacitors or fix capacitors and vary g_m's

Some leapfrog properties



What can be said about sensitivities of parameters such as band edges of leapfrog filters? If these sensitivities are not at or near 0, are they at least very small?

No! Nothing can be said about these sensitivities and they are not necessarily any smaller than what one may have for other structures such as cascaded biquads

Instead of having components (such as L's or C's) in the image of the lossless ladder network there are circuits such as integrators or biquads with more than one characterization parameters. Are the sensitivities of $|T(j\omega)|$ to these components also 0 at frequencies where the "parent" passive filter are 0?

Yes! The following theorem addresses this issue in the case of integrators

Theorem: If f(u) is a function of a variable u where $u=x_1x_2$, then

$$S_u^f = S_{x_1}^f = S_{x_2}^f$$

Note: Although the results are the same as for the sensitivity of kf, in this case both x_1 and x_2 are variables whereas in the former case k is a constant.

As a consequence, if the unity gain frequency of an integrator which may be expressed (for example) as 1/RC, the transfer function magnitude sensitivity to both R and C vanish at frequencies where the sensitivity to I_0 vanishes

Stay Safe and Stay Healthy !

End of Lecture 30